TRANSFER PROCESSES IN A FREE (JET) TURBULENT BOUNDARY LAYER

L. A. VULIS, T. P. LEONT'EVA, Z. B. SAKIPOV, I. B. PALATNIK and B. P. USTIMENKO Institute of Energetics of the Academy of Science of the Kazakh S.S.R., U.S.S.R.

(Received 23 August 1961)

Аннотация—Излагается метод решения тепловых и динамических задач струйных течений путём сведения дифференциальных уравнений пограничного слоя к эквивалентному уравнению теплопроводности. Приводится физическое обоснование метода, сводящееся к переносу процесса в фиктивное линейное пространство, связь которого с реальным полем течения определяется из опыта по зависимости скорости на оси струи от координаты.

Излагаются результаты специально поставленных опытов по изучению механизма смешения неизотермических струй сжимаемого газа. Результаты показывают определяющую роль градиента плотности потока импульса в процессах переноса в турбулентных струях.

(1) PHENOMENA of turbulent transfer in free flow of an incompressible liquid or compressible gas form the working basis for different technical devices. They are extremely important in furnaces, stove techniques, industrial ventilation, etc. For this reason the experimental and theoretical study of heat, matter and impulse transfer processes in various jet flows is of considerable interest.

Because of insufficient development of the general theory of turbulent flow, different semiempirical methods have become widely used in studying turbulent jets. One of these is the calculation of free turbulent flow by substituting boundary layer differential equations for equations of heat conductivity. This method, suggested several times in [1, 2, etc.], was widely developed in investigations carried out at the Institute of Energetics of the Academy of Science of the Kazakh S.S.R. and at the Kazakh State University [3, 5]. These investigations were concentrated on such flows as submerged jets of a finite dimension, ets in concurrent and counter flows, etc. Considerable attention was paid to the study of the mixing mechanism in turbulent flow.

(2) The method being developed for solving jet problems is connected with both the external resemblance of integral curves of velocity distribution in a jet and with temperature of a heat source in a solid.

The successful application of this method depends on the fulfillment of certain conditions. Firstly, the flow must depend on the right class of boundary layer problems (i.e. when the transverse velocity component is small compared with the longitudinal component, and the longitudinal transfer is small compared with the transverse transfer). Secondly, motion must take place well away from solid walls and be practically isobaric (i.e. flow lines relatively straight). Thirdly, the initial impulse distribution (heat content), may be arbitrary and this is one of the significant advantages of this calculation scheme.

Under these conditions the phenomenon of jet propagation, from the physical point of view, is reduced to level initial disturbances (impulse, heat flow, etc.).

The jet propagation process is described by non-linear boundary layer differential equations. Although at a distance from the mouth the flow attains steady state, and motion and energy equations are reduced to ordinary differential equations, the transition to steady state motion will be non-linear and dependent on initial conditions. Let us assume that it is possible to bring about the deformation of a real space (x, y) into a fictious space (τ, φ) where the process of jet dispersion may be described by linear differential equations.

As further investigations showed, in jet problems it is sufficient to deform only the longitudinal co-ordinate x.

In this case heat and impulse transfer equations in the new deformed space will take the form:

$$\frac{\partial u}{\partial \tau_i} = \frac{1}{v^k} \frac{\partial}{\partial v} \left(y^k \frac{\partial u}{\partial v} \right). \tag{1}$$

Here k=0 and corresponds to a flat jet; k=1, to the axisymmetric jet; i=q; $u=\rho u^2$ corresponds to the dynamic and $i=\tau$; $u=\rho u c_p \Delta T$, to the heat problem.

The law of transformation of values of $\tau_d(x)$ and $\tau_h(x)$ is not known in advance but may be obtained from comparison with experimental results, e.g. by comparing calculation and experimental data for the change along the axis of a jet.

In a dynamic problem the co-ordinate $\tau_h(x)$ differs from the corresponding dependant $\tau_d(x)$ because of the difference in thickness of dynamic and thermal mixing regions.

In the steady state region of motion proceeding from dimensionality for $\tau_d(x)$ and $\tau_h(x)$ we obtain:

$$\tau_d = c_d x^2, \tag{2}$$

$$\tau_h = c_h x^2. \tag{3}$$

In this case the ratio

$$\frac{\tau_d}{\tau_b} = \frac{c_d}{c_b} = \sigma_t \tag{4}$$

indicates the turbulent Prandtl number. As further investigation showed (Fig. 1), the value σ_t determined by equation (4) coincides with the numerical value for the ordinary Prandtl number.

Thus, in the steady state region as well as in the theoretical "mixing lengths" it is necessary to determine only one experimental constant. The developed equivalent heat conductivity problem method makes it possible to calculate the continuous deformation of velocity and temperature profiles with arbitrary initial distribution.

Thus, we succeeded in solving a number of problems (e.g. jet of a finite dimension, a system of jets, etc.), whose solution by other methods is cumbersome or completely impossible.

(3) Now let us consider problems of propagation of submerged jets with a complicated initial velocity (temperature) distribution.

Referring to [3-5], we shall dwell only on the following:

Table 1 gives the basic types of jets investigated, initial conditions and formulae for calculating velocity and temperature fields

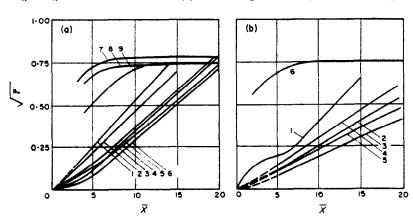


Fig. 1. Dependence $\tau(x)$ for jets investigated:

(a) $1-\sqrt{\tau}$; $2-\sqrt{\tau}_d$ —flat jet $(\rho u_0^2=\text{const})$; $3-\sqrt{\tau}_h$; $4-\sqrt{\tau}_d$ axisymmetrical jet $(\rho u_0^2=\text{const})$; $5-\sqrt{\tau}_h$; $6-\sqrt{\tau}_d$ axisymmetrical jet $\{\rho u_0^2\sim [1-(\gamma/r_0)]^{2/7}\}$; $7-\overline{\tau}_d/\tau_h$ flat jet; $8-\overline{\tau}_d/\tau_h$ —axisymmetrical jet $(\rho u^2=\text{const})$; $9-\tau_d/\overline{\tau}_h$ —axisymmetrical jet $\{\rho u_0^2=[1-(\gamma^0/r_0)]^{2/7}\}$.

(b) 1—two concurrent flat jets $[(u_e/u_1) = 2]$; 2,3,4,5—jet in a concurrent flow where 2— $\sqrt{\tau}_d$ $[(u_e/u_i) = 0.143]$; 3— $\sqrt{\tau}_d$ $((u_e/u_i) = 0.295)$; 4— $\sqrt{\tau}_h$ $[(u_e/u_i) = 0.143]$; 5— $\sqrt{\tau}_d$ $[(u_e/u_i) = 0.295]$; 6— τ_d/τ_h for jet in concurrent flow. $\tau = \tau/e$; $\overline{x} = x/e$ (e—is $\frac{1}{2}$ width of nozzle for flat jet, diameter of nozzle for axisymmetrical jet).

Table 1

Kind of jet	Initial conditions	Solution Flow scheme
Flat jet	$ y < b$; $(\rho u^2)_0 = \text{const}$; $(\rho u c_p \Delta T)_0 = \text{const}$; $ y > b$; $(\rho u^2)_0 = 0$	$\tilde{u} = \frac{1}{2} \left[\psi \left(\frac{y+b}{2\sqrt{\tau}} \right) - \psi \left(\frac{y-b}{2\sqrt{\tau}} \right) \right];$
Two flat con- current jets of different velocity	$\begin{array}{lll} -\infty &< y &< b_2; \; (\rho u^2)_0 &= 0; \\ -b_2 &< y &< 0; & \rho u^2 &= (\rho u^2)_2; \\ 0 &< y &< b_1; \; \rho u^2 &= (\rho u^2)_1; \\ b_1 &< y &< \infty; \; \rho u^2 &= 0. \end{array}$	$ \tilde{u} = \frac{1}{2} \left[\psi \left(\frac{y}{2\sqrt{\tau}} \right) - \psi \left(\frac{y - b_1}{2\sqrt{\tau}} \right) + \frac{1}{2} \frac{(\rho u^2)_2}{(\rho u^2)_1} \left[\psi \left(\frac{y + b_2}{2\sqrt{\tau}} \right) - \psi \left(\frac{y}{2\sqrt{\tau}} \right) \right] $
Two flat con- current jets at different distances from each other	$\begin{array}{lll} -\infty & < y < z; & \rho u^2 = 0; \\ -z & < y < d; & \rho u^2 = {\rm const}; \\ -d & < y < d; & \rho u^2 = 0; \\ d & < y < z; & \rho u^2 = {\rm const}; \\ z & < y < \infty; & \rho u^2 = 0. \end{array}$	$ \tilde{u} = \frac{1}{2} \left[\psi \left(\frac{y + Z}{2\sqrt{\tau}} \right) - \psi \left(\frac{y + b}{2\sqrt{\tau}} \right) + \psi \left(\frac{y - b}{2\sqrt{\tau}} \right) - \psi \left(\frac{y - Z}{2\sqrt{\tau}} \right) \right] $
Axisymmetrical jet	$0 < y < r_0; (\rho u^2)_0 = \text{const} (\rho u c_2 \Delta T)_0 = \text{const}; y > r_0; \rho u^2 = 0.$	$\bar{u} = \frac{1}{2\pi\tau} \int_{0}^{\tau_0} \rho d\rho \int_{-\pi/2}^{\pi/2} \exp\left[-(\rho^2 + y^3 - 2y\rho\sin\varphi)/4\tau\right] d\varphi$
Axisymmetrical jet	$ y < r_0;$ $(\rho u^2)_0 \sim [1 - (y/r_0)]^{2/7}$ $(\rho u c_p \Delta T)_0 = \text{const};$ $ y > r_0;$ $\rho u^2 = 0.$	$\tilde{u} = \frac{1}{2\pi\tau} \int_0^{\tau_0} \left(1 - \frac{\rho}{r_0}\right)^{2/7} \rho d\rho$ $\int_{-\pi/2}^{\pi/2} \exp\left[-(\rho^2 + y^2 - 2y\sin\varphi)/4\tau\right] d\varphi$
Jet flowing from two concentric nozzles	$r_2 < y < r_1; (\rho u^3)_0 = (\rho u^2)_2;$ $r_1 < y < 0; (\rho u_2)_0 = [1 - (y/r_0)]^{2/7};$ $ y > r_2; \rho u^2 = 0.$	$\bar{u} = \frac{1}{2\pi\tau} \int_0^{\tau_1} \left(1 - \frac{\rho}{r_0}\right)^{2/7} \rho d\rho$ $\int_{-\pi/2}^{\pi/2} \exp\left(\rho^2 + y^2 - 2y\rho\sin\varphi/4\tau\right) d\varphi$ $+ \frac{1}{2\pi\tau} \frac{(\rho u^2)_2}{(\rho u^2)_1} \int_{-\pi/2}^{\pi/2} \exp\left(\rho^2 + y^2 - 2y\rho\sin\varphi/4\tau\right) d\varphi$

determined with the solution of equation (1). The symbols used here are as follows:

$$\tilde{u} = \frac{\rho u^2}{(\rho u^2)_0}$$

or

$$\bar{u} = \frac{\rho u c_p \Delta T}{(\rho u c_p \Delta T)_0}$$

represent the dynamic or heat problem respectively; $(\rho u^2)_0$; $(\rho u c_p \Delta T)_0$ are initial values of impulse density flow and heat content; b_i is the nozzle half width for a flat jet flow; r_i is the nozzle radius for an axisymmetrical jet flow; L is the distance between nozzles for two flat jet flow.

$$\varphi(z) = \frac{2}{\sqrt{(\pi)}} \int_{0}^{z} \exp(-t^{2}) dt$$

is the error integral; φ is the variable angle of integration for sources in a cylindrical system of co-ordinates.

In the cases considered the experimental flow investigation was carried out on special stands, attention being centred mainly on obtaining a detailed picture of flow on the basis of analysis of velocity and temperature fields. For the theoretical solution hydrodynamic and hydrostatic integrators were widely used, application of this method to the solution of jet problems is described in [6, 7].

Regime and geometrical characteristics of the flows investigated are given in Table 2.

Table 2

Kind of flow	$r_1(b_1)$ (mm)	$r_2(b_2)$ (mm)	<i>u</i> ₁ (m/s)	<i>u</i> ₂ (m/s)	u_1/u_2	z ₁ (mm)	$\begin{array}{c} \Delta T_0 \\ (T_0 - T_{\text{average}}) \end{array}$
Flat jet	8-4		39-0	_			10–15°
Two flat concurrent jets of different velocity	10	10	77·6; 74·3; 35·0; 15·2	38·8; 26·5; 17·5; 30·4	0·5; 2·0; 2·8	0	
Two flat concurrent jets at different distances from each other	7∙0	7.0	80∙8	80.0	1	34·8 25·5 17·5 3·5	
Axisymmetrical jet with uniform initial velocity distribution	5-0	_	60-0				20–25°
Axisymmetrical jet with initial velocity distribution of type	6-3		40.0	404444	_	*****	20–25°
Axisymmetrical jet flow- ing from two con- centric nozzles	20-75 20-75 20-75 20-75 20-75 20-75 20-75 20-75 20-75 20-75 20-75 2-0 2-0	39·25 39·25 39·25 53·0 53·0 67·0 67·0 67·0 6·75 6·75		_	1·0 2·0 3·0 1·0 2·0 3·0 1·0 2·0 3·0 1·0 2		

Figure 1 shows dependents $\tau_d(x)$ and $\tau_h(x)$ for the flow cases investigated. Velocity and temperature profiles in transverse sections of the jets were calculated by these dependents. The calculation and experimental data are in good agreement.

(4) The application of the developed calculation method of jet problems to complicated turbulent flows, in particular, to a jet propagating in a concurrent flow is of considerable interest.

A detailed discussion of this problem is given in [8]. It is worthwhile to note that because of linearity and uniformity of equation (1) it may be easily generalized for motion of a surrounding medium with a constant velocity. In this case value u in equation (1) should mean $\rho u^2 - (\rho u^2)_f$ or $\rho u c_p \Delta t$ for dynamic and heat problems respectively. Here $(\rho u^2)_f$ is the value of impulse density flow in a surrounding flow.

Although formally the complicated flow problem is reduced to corresponding problems for a submerged jet, differences in boundary conditions leading to boundary distortions, etc. influence the type of dependent values τ_h and τ_d upon x (Fig. 1b). Otherwise the solution of the problem is exactly the same as the solution for the submerged jet under equal initial conditions (see Table 1).

The distribution of a slightly heated jet in a concurrent uniform flow was investigated experimentally. Regime and geometrical characteristics of the flow investigated are given in Table 3.

Dependents $\tau_d(x)$ and $\tau_h(x)$ obtained in an analogous way to that described above are shown in Fig. 1(b). Comparison of the experimental and calculation data obtained for impulse and temperature flow density distribution

Ta	hle	3

d ₀ jet (mm)	d ₀ jet (m/s)	<i>u_f</i> (m/s)	$\frac{u_f}{u_i} = m$	∆T ₀ jet (°C)	
20	60	0	0	20–25	
20	42	6	0-143	20-25	
20	42	12-4	0.295	20-25	
20	42	15.8	0.380	20-25	

in transverse jet sections (Fig. 2) showed their satisfactory convergence.

(5) Another type of complicated turbulent flow studied is the jet propagation in a counter uniform flow. Such flows are often met in practice, and apparently may find a wide application as aerodynamic flame stabilizers [9, 10].

While referring for details to [9, 11, 12, 13] we noted the following.

The type of flow considered has some peculiarities which make it different to ordinary problems of boundary layer theory. In particular, in such a flow there is a considerable static pressure gradient (up to 30 per cent from the initial jet head velocity) both in its longitudinal and transverse directions. In addition, the

transverse velocity component which at the end of the jet greatly exceeds the longitudinal component becomes a considerable value.

Hence, the calculation carried out by the method of substitution of the equivalent heat conductivity problem in this case gives only a correct qualitative flow picture (presence of zero surface velocities, closed circulation zones, etc.) but does not give any satisfactory quantitative agreement with experiment. In the work carried out, fields of static pressure, velocity and excessive temperature (above the surrounding flow) at different values of the basic parameter $m = (u_f/u_{0f})$ were experimentally studied.

Regime and geometrical characteristics found in experiments when studying a flat counter jet are listed in Table 4.

(6) The investigation of heat and mass transfer, etc. in a jet is of considerable interest for studying flows. In a number of works (see, e.g. Ref. 6) it was assumed that impulse flow density gradient ρu^2 plays a determinative role in transfer processes. To verify this assumption special experiments were carried out in which the attenuation of a compressed gas

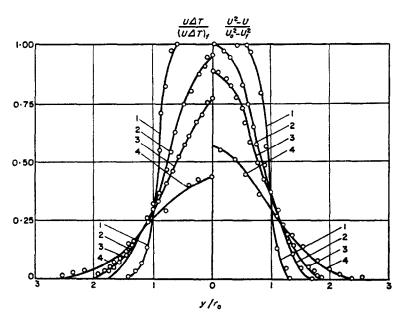


Table 4

Nozzle width (mm)	Regime m	1	2	3 0·3	4 0·35	5 0·40	6	7	8
		0.2	0.25				0-45	0.5	
6	u,	8.0	10-0	9.0	14.0	16-0	18.0	16.0; 25.0	20.0
	u_{0j}	40-0	40-0	30-0	40-0	40.0	40-0	32.0; 50.0	33-3
12	u,	8.0		9.0				10.0; 16.0; 25.0	20.0
	u_0 ,							20.0; 32.0; 50.0	33-3
16	u _t			9.0	·	16.0	18∙0	10.0; 16.0; 25.0; 30.0	20.0
	u _n ,			30.0		40.0	40.0	20.0; 32.0; 50.0; 60.0	33-3
	ΔT_0			22.0		22.0		12.0; 22.5; 29.5; 12.0	
	-							22·0; 36·0 36·0	18-0

Table 5

Jet				Concurrent flow			Relation		
<i>T</i> ₀ (°K)	<i>u</i> ₀ (m/s)	$\frac{\rho_0 \mu_0^2}{2}$ (mm water column)	T₀° ^K flow	<i>u,</i> (m/s)	$\frac{\frac{\rho_f u_f^2}{2}}{2}$ (mm water column)	$\frac{T_0}{T_f} = w$	$\frac{u_f}{u_0}=m_f$	$\frac{(\rho u^2)}{(\rho u^2)_0} = m$	
320	17-60	16-200	300	0-29-5	0-48	1.08	0–1·67	0-3	
600	24-87	16-200	300	0-29-5	0-48	2.0	0-1-22	0–3	
900	30–105	16–200	300	0-29-5	0–48	3.0	0-1-0	0-3	

propagating in a concurrent uniform flow was investigated.

Limits of the change of parameters of a jet and flow as well as their relations are given in Table 5.

As a basic value characterizing transfer intensity in a jet, the relation of temperature exceeding the surrounding flow at a fixed point on the axis of the jet to initial temperature of the jet $(\Delta T_m/\Delta T_0)$ was chosen.

The change of this relation at various ratios of jet and flow dynamic heads gives a vivid representation of transfer intensity in a jet.

Figure 3 shows the change of this relation depending on the parameter m (see Table 5) for the greatest parameter value w = 3 obtained in our experiments. As is seen from this figure, the maximum value of $(\Delta T_m/\Delta T_0)$, and, consequently, the minimal mixing practically coincide with the value m = 1 (at equal values of dynamic

heads in a jet). Relation of velocity values (u_f/u_0) and mass velocity $(\rho u)_f/(\rho u)_0$, are also given for comparison along the abscissa axis.

As is seen from this figure, at equal velocity values in the jet and the flow or at those of the mass velocity the mixing intensity is noticeably higher than at equal values of the dynamic heads.

The experimental results obtained confirm the assumption about the determinative role of the impulse flow density gradient in transfer processes and also substantiated the choice of impulse flow density, heat content, etc.

(7) In obtaining transfer processes in turbulent jets, the so-called turbulent Prandtl number σ_t determined as the relation of turbulent impulse and heat transfer coefficients $[\sigma_t = (\nu_t/a_h)]$ plays a significant role.

Knowing the above mentioned number allows the calculation of a heat problem by known

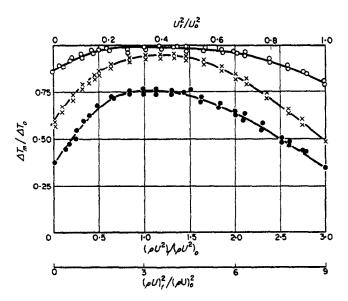


Fig. 3. Dependence $\Delta T_m/\Delta T_0$ on $(\rho u^2)_1/(\rho u^2)_0$ [as well as on u_1^2/u_0^2 and $(\rho u)_1^2/(\rho u_0^2)$] $0 - x/d_0 = 2.5$; $x - x/d_0 = 5$; $0 - x/d_0 = 7.5$.

results obtained theoretically or experimentally for a dynamic problem.

As is known from the literature, the number σ_t has the value $\sim 0.72-0.75$ in free turbulent jets. However, the determination of this number was carried out in a great majority of the cases with an air motion where this number is near to the physical meaning of the Prandtl number $[\sigma = (\nu/a) = 0.72]$.

In this direction special experiments for determining σ_t in submerged axisymmetrical slightly heated turbulent jets of high viscosity at the physical Prandtl number of $\sim 10^3$ (transformer oil) were carried out at the Institute of Energetics of the Academy of Science of the Kazakh S.S.R. Fields of the dynamic head and temperature in various jet sections were studied in the experiments.

To determine σ_t the relation between temperature distribution and velocity distribution was used from the known free turbulence theory

$$\frac{\Delta T}{\Delta T_m} = \left(\frac{u}{u_m}\right)^{\alpha_t}.$$
 (5)

The value σ_t determined from experimental data with the help of formula (5) proved to be 0.72 and constant along the jet section.

Thus, value σ_t is confirmed as a purely hydrodynamic characteristic of the flow and does not depend upon the physical properties of the liquid.

REFERENCES

- N. REICHARDT, Gesetzmassigkeiten der freien Turbulenz, Forschungsh Ver. Disch. Ing., 414, (1942).
- 2. P. V. MELENT'EV, Nauchnye trudy Stalingradskogo mekhanicheskogo instituta, tom 1, (1952).
- L. A. Vulis, Izv. An Kazakh. S.S.R., seria energeticheskaya, N 2, 18 (1960).
- B. P. USTIMENKO, Izv. AN Kazakh. S.S.R., seria energeticheskaya, N 2, 18 (1960).
- I. B. PALATNIK, Izv. AN Kazakh. S.S.R., seria energeticheskaya, N 2, 18 (1960).
- Sbornik, Issledovanie fizicheskikh osnov rabochego protsessa topok i pechei (Edited by L. A. VULIS), Izdatel'stvo AN Kazakh. S.S.R. (1957).
- V. G. BESPALOVA, Izv. AN Kazakh. S.S.R., seria energeticheskaya, N 2, 16 (1960).
- I. B. PALATNIK, Izv. AN Kazakh. S.S.R., seria energeticheskaya, N 2, 18 (1960).
- L. A. Vulis and T. P. Leont'eva, Izv. AN Kazakh. S.S.R., seria energeticheskaya, N 9 (1955).
- L. A. VULIS, T. P. LEONT'EVA and A. V. TONKONOGY, Vestnik AN Kazakh. S.S.R., N 5 (1954).
- L. A. Vulis, V. P. Kashkarov and T. P. Leont'eva, Issledovanie fizicheskikh osnov rabochego protsessa topok i pechei, *Izdatel'stvo AN Kazakh*. S.S.R. (1957).
- 12. T. P. LEONT'EVA and B. P. USTIMENKO, Trudy Instituta energetiki AN Kazakh. S.S.R., tom III.
- A. SCHAFFER and A. B. CAMBEL, Jet Propulsion, 25, (1955); 26 (1956).

Abstract—The method of solution of heat and dynamic problems for jet flows is given by means of reduction of boundary layer differential equations to an equivalent heat conduction equation. A physical substantiation of the method is presented, reduced to the transfer of the process into a fictitious linear space, the connexion of which with the real field of a flow is determined experimentally according to the dependence of the velocity at the axis of the jet on the co-ordinate.

Results of special experiments on studying the mixing mechanism of non-isothermal jets of a compressible gas are adduced. They show the determinative role of the impulse flow density gradient for transfer processes in turbulent jets.

Résumé—Cet article présente une méthode de résolution des problèmes dynamique et thermique dans les jets, elle consiste à réduire les équations différentielles de la couche limite en une équation de conduction thermique équivalente.

L'auteur donne une vérification physique de la méthode en transposant son problème dans un espace linéaire fictif. La relation entre cet espace et le champ réel de l'écoulement est déterminée expérimentalement d'après la dépendance de la vitesse axiale du jet sur l'autre coordonnée.

Les résultats d'expériences particulières sur le mécanisme du mélange de jets non-isothermes de gaz compressible sont présentés. Ils montrent le rôle déterminant du gradient de densité sur les phénomènes de transport dans les jets turbulents.

Zusammenfassung—Als Lösungsmethode wärmetechnischer und dynamischer Probleme des turbulenten Freistroms wird eine Verkürzung der Differentialgleichungen der Grenzschicht in äquivalente Gleichungen für Wärmeleitung angegeben. Um die physikalische Gültigkeit der Methode zu beweisen, wird der Prozess in einen fiktiven linearen Raum übertragen und die Verbindung mit dem wirklichen Strömungefeld experimentell hergestellt; dabei soll die Geschwindigkeit in der Achse von der Koordinate abhängen.

Die Ergebnisse spezieller Versuche über den Mischmechanismus nicht-isothermer Strahlen eines kompressiblen Gases sind angegeben. Sie zeigen den massgebenden Einfluss des Gradienten der Impulsstromdichte auf Übertragungsvorgänge im turbulenten Freistrom.